aberrations. The results are summarized in Table 4. The MRT algorithm shows larger errors in the order of a small fraction of a micrometer.

Table 4. Performance of the four algorithms on a simulated corneal surface. The error in μm of the non-rotationally symmetric aberration is evaluated.

	RMS of dominant feature	FRT	MRT	Basic AS	SREC AS
RMS astigmatism	0.7498	$< 1 \times 10^{-15}$	0.0819	−0.0150	0.0008
RMS trefoil	0.1717	$< 1 \times 10^{-15}$	−0.0439	0.0019	0.0030
RMS quadrafoil	0.1808	$< 1 \times 10^{-15}$	−0.0296	0.0017	0.0015

Figure 3 shows residual curvature plots of the different algorithms in reconstructing the simulated corneal surface. The residual curvature is calculated by taking the absolute difference of the curvature map of the simulated surface and the curvature map of the reconstructed surface by the algorithm. The FRT algorithm produced a flat residual curvature map (negligible error). The largest local curvature was produced by the MRT algorithm. The arc step algorithm has an improved performance compared to the MRT algorithm. This performance was further improved by the SREC AS algorithm.

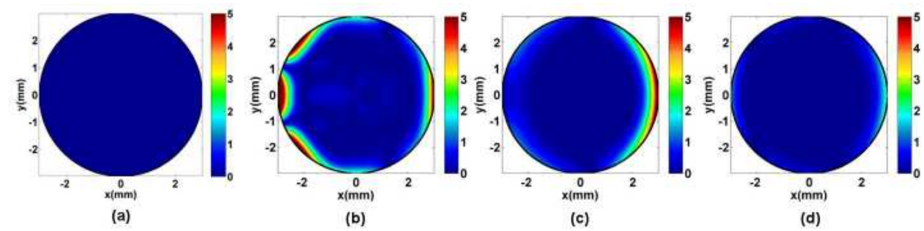

Fig. 3. Residual curvature map in Diopter for a simulated cornea with no abnormalities. Here (a) represents the reconstruction by the FRT algorithm, (b) is the reconstruction by MRT algorithm, (c) is the reconstruction by the Basic AS algorithm and (d) is the SREC AS algorithm.

4.3 Effect of noise

So far the quantitative analysis is done without the presence of noise in the system. In actual measurements the presence of noise is inevitable. Many factors contribute to this, i.e. eye movements, shadows created by nose and eye-lashes, pixel resolution of the camera, fluctuations of the light intensity captured by the camera, alignment precision of the instrument to name a few. Noise will have an effect on the performance of corneal surface reconstruction algorithms. This was first mentioned by Klein [11], stating that one-to-one correspondence corneal topography surface reconstruction described by Halstead [16] has greater robustness to noise than arc step algorithms. This was confirmed recently by Sokurenko [20]. We also implemented noise simulation in this study. Gaussian noise was applied to the image points (at a modified camera plane such that there is unit magnification between the camera plane and the corneal apex plane) and the amplitude was varied taking values of 0, 1, 2, 4, 6, 8 and 10 μm. The range from 0 to 10 μm amplitude noise was chosen to account for typical camera resolution of corneal topography systems [19,20,27]. This range is on the conservative side because this does not include effects of eye movements and other factors that contribute to noise. Another important consideration is the fact that for Placido-based corneal topography, the edge detection of the rings has a big effect. For this case, subpixel accuracy can be achieved in the direction along the ring but there would be more uncertainties in the radial direction (perpendicular to the ring) [11,19], therefore the appropriate noise distribution in this case would be radially oriented, as it is applied in this study. For the SREC AS algorithm, only the performance for 0 and 1 μm noise is reported

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because the algorithm is unstable for noise ≥ 2 µm. The noise simulation was done for 100 realizations. The performance of the four algorithms were assessed in terms of accuracy (the difference between the mean value of the measurements and the actual value) as represented by the height of the bars in the figures and precision (the spread expressed in standard deviation of the measured values) as represented by the error bars in the figures.

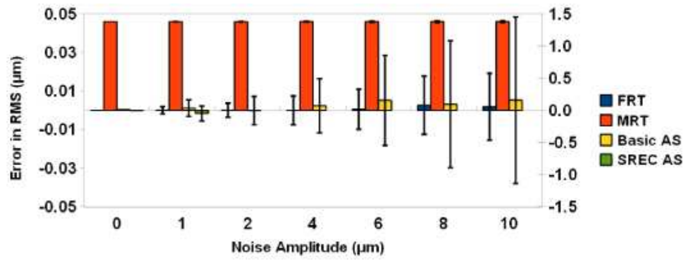

Fig. 4. Astigmatism error in RMS (up to Zernike order 8) of artificial surface. The results for the MRT algorithm are plotted with respect to the secondary y-axis.

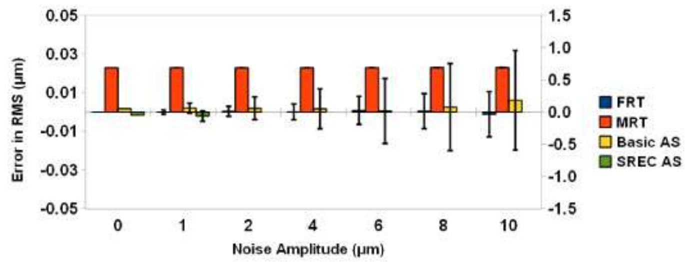

Fig. 5. Trefoil error in RMS (up to Zernike order 8) of artificial surface. The results for the MRT algorithm are plotted with respect to the secondary y-axis.

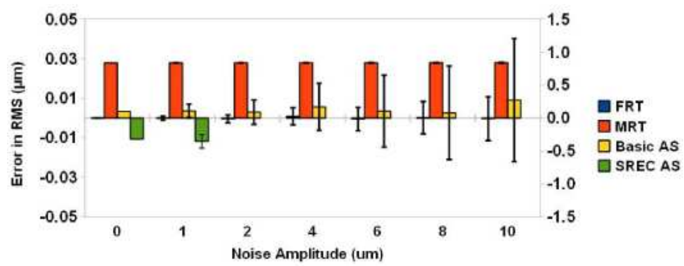

Fig. 6. Quadrafoil error in RMS (up to Zernike order 8) of artificial surface. The results for the MRT algorithm are plotted with respect to the secondary y-axis.

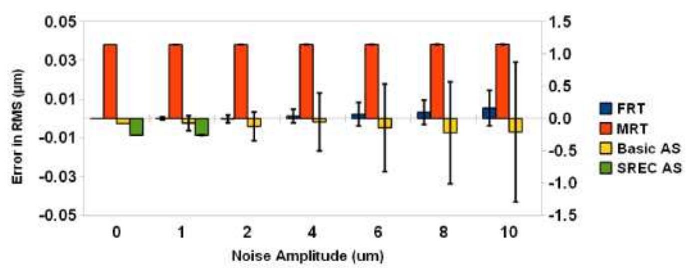

Fig. 7. Octafoil error in RMS (up to Zernike order 8) of artificial surface. The results for the MRT algorithm and the SREC AS algorithm are plotted with respect to the secondary y-axis.

Figure 4-Fig. 7 show the effect of noise in the performances of the different algorithms in reconstructing the toric and multifoil surfaces. The results show a consistent trend that the arc

step algorithms are more affected by noise. The spread in the RMS error values are about 2 times greater for the arc step algorithms than the FRT-based algorithms. Although the MRT algorithm produces the largest error, this algorithm is as robust as the FRT algorithm with respect to noise. The spread due to noise for these two systems are almost equal in magnitude.

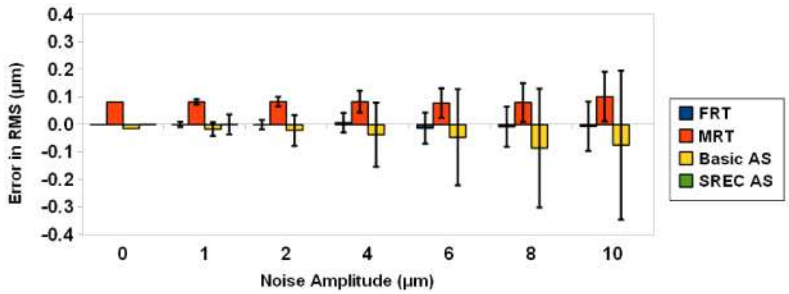

Fig. 8. Astigmatism error in RMS (up to Zernike order 8) of normal eye.

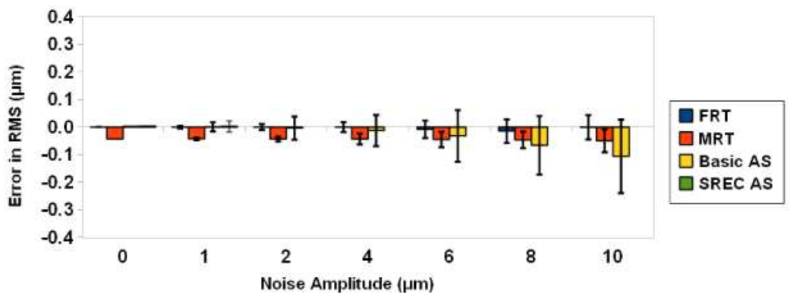

Fig. 9. Trefoil error in RMS (up to Zernike order 8) of normal eye.

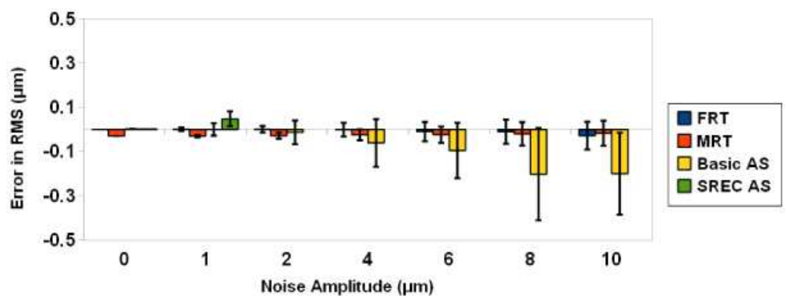

Fig. 10. Quadrafoil RMS error (up to Zernike order 8) of normal eye.

Figure 8-Fig. 10 show the effect of noise in the performances of the different algorithms in reconstructing a typical cornea with no abnormalities. The precision of the arc step algorithms are 2-3 times worse compared to that of the arc step algorithms. Furthermore, as the noise amplitude increases there is also degradation of accuracy for the arc step algorithms. For quadrafoil in particular the accuracy and precision is in the order of quarter of Diopter (~0.33 μm RMS error).

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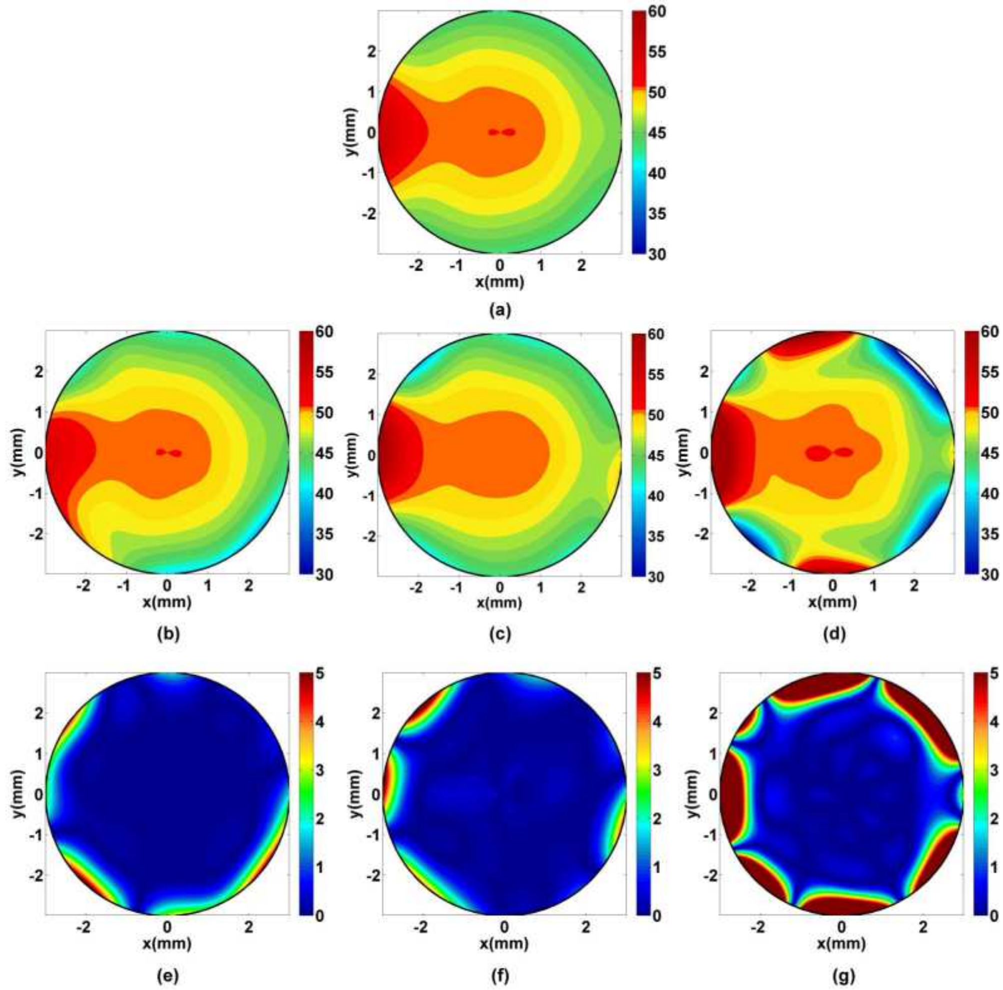

Fig. 11. Curvature maps (a,b,c,d) and residual curvature maps (e,f,g) (in Diopter) of the reconstruction of a cornea with no abnormalities by different algorithms at a noise level of 10 micrometer. The simulated reference surface is represented by (a). The algorithms used for reconstruction are FRT (b,e), MRT (c,f) and Basic AS (d,g). The SKEC AS algorithm was excluded here because of numerical instability at this noise level. The curvature isolines of the curvature maps a,b,c and d are 6/5th diopter apart, and the curvature isolines of the residual curvature maps e, f and g are 1/8th diopter apart.

As a typical example the curvature maps and corresponding residual curvature maps for a reconstruction of a simulated cornea at a noise level of 10 micrometer are depicted in Fig. 11. The FRT algorithm performs well under these conditions remaining stable well beyond the 4 mm corneal zone region. The MRT algorithm is also relatively stable in the peripheral region but suffers from large artifacts in the central region. The astigmatism as manifested in the bow-tie structure in the actual curvature map disappeared. Finally, the basic arc step algorithm is unstable beyond the 4 mm corneal zone region resulting in residuals of several diopters. Also in the central region some artifacts can be seen in the residual curvature map exceeding 1 diopter appear. The basic arc step curvature map shows an increase magnitude of astigmatism as manifested by the larger bow tie structure. Also, the appearance of a quadrafoil-like artifact is apparent from the orange curvature isolines in Fig. 11d.

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5. Discussion

In this study the performance of four surface reconstruction algorithms for two types of corneal topography system were evaluated under different experimental conditions, by varying the amount of noise. First the case without any noise is discussed where all experimental conditions are assumed to be perfect and the individual algorithm can be evaluated. It is known that tilt and decentration affect measurements of corneal surfaces [28,29]. However, this aspect was not yet included in the analysis so that a simple and clear description of the effects of skew ray error can be made. The extension of the analysis to include tilt and decentration can be part of future investigations.

5.1 Simulations without noise

Out of the four algorithms discussed here, the MRT algorithm is by far the least accurate as can be seen from Table 3, Table 4 and Fig. 3. This demonstrates the danger of not taking into account the skew ray error. On the other hand, the arc step algorithms (Basic AS and SREC AS) are quite accurate in reconstructing the corneal surface. The procedure in arc step algorithms where it is ensured that first and second derivatives are continuous for the whole corneal surface partially corrects for the skew ray error and reduces the error to negligible amounts. It is also notable that the SREC AS algorithm reduces the error of the basic arc step algorithm and this is demonstrated clearly in Fig. 3. However, this does not happen on a consistent basis as can be seen from the reconstruction of the octafoil surface (Table 3) and also the reconstruction of the trefoil component of the simulated cornea (Table 4). We should note that previous demonstrations of the SREC AS algorithm were done in a telecentric CT system while in this study it was implemented in a nodal point system. This could explain why results of previous studies [8,10] were limited to cases where SREC AS algorithm improved the performance of the basic arc step algorithm.

Although the error of the arc step algorithms is small, it is 11-12 orders higher than that of the FRT algorithm. This is consistent with the results of Sokurenko [20] where it was noted that the error of point to point topography was 7-9 orders lower than Placido-based topography. The difference in the order of magnitude observed could be due to differences in the numerical tolerance of the algorithms used, in principle the results show that the accuracy is only limited by the machine precision and noise.

5.2 Simulations with noise

There are two notable effects of noise in the performance of corneal surface reconstruction algorithms: the effect in precision and the effect in accuracy.

With regard to precision, the point to point algorithms (FRT and MRT) are least affected by noise compared to their arc step algorithm counterparts. This is true for both artificial surfaces and the simulated corneal surface. The precision of all four algorithms measured for artificial surfaces are quite small (< 0.05 μm). The precision measured for the simulated cornea is better than 0.1 μm (0.08 eq D) for the point to point corneal topography algorithms and can reach as high as 0.27 μm (0.21 eq D) for the basic arc step algorithm. If we take 0.125 diopter as a threshold for clinical relevance [30,31], then we see that the precision of the point to point algorithms even at a noise amplitude of 10 μm is still tolerable while that of the basic are step algorithm is clinically relevant. It appears that noise interferes with the inherent skew ray error correction mechanism of arc step algorithms. The effect is so great in the SREC AS algorithm that at a noise level of 2 μm the algorithm is already unstable. The SREC AS algorithm [8,11] attempts to improve upon the earlier published arc step algorithms [6] by ensuring that the second derivative, both in radial (along the meridian) and angular (along the ring) direction, is continuous. The results suggest that this will work with no noise in the measurements. In reality, because noise is always present in measurements the performance of the SREC AS algorithm seems to deteriorate quickly which makes it likely

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that the algorithm crashes even at a 2 μm noise level. There is also disparity between the precision obtained from artificial surfaces compared to the simulated cornea. This suggests that a good performance in measurement of artificial surfaces does not guarantee a good performance for real corneal surfaces.

In terms of accuracy the FRT algorithm still gives a good performance for all cases even at 10 μm noise level. The worst accuracy was 0.028 μm (0.022 eq D) in reconstructing the quadrafoil aberration of a simulated corneal surface. On the other hand, the accuracy of the arc step algorithms deteriorated with noise and the magnitude of the inaccuracy becomes greater the more complex the non-rotationally symmetric aberrations become. For the basic arc step algorithm, with increasing aberration complexity (astigmatism, trefoil, quadrafoil) the absolute accuracy at 10 μm noise level was 0.075 μm (0.058 eq D), 0.106 μm (0.082 eq D), 0.200 μm (0.154 eq D) respectively. The error in quadrafoil aberration is higher than the clinically tolerable level of 0.125 D and therefore clinically relevant. It is also notable that although the MRT algorithm is inaccurate without noise, the accuracy remains stable in the presence of noise showing that the precision of point to point algorithms is not significantly affected by skew-ray errors.

5.3 Clinical implications

It is known that edge artifacts are present in corneal topographic maps when the surface is modeled by Zernike polynomials [3]. Apart from edge artifacts that cover less than 10% of the evaluation area, the residual curvature map of the simulated cornea (see Fig. 11) when FRT is implemented at 10 μm noise level is relatively flat. This is not true for the arc step algorithm. The area of the edge artifact is covering a larger area (about 30%) and the central region does not display a flat residue. There are several patches where the error is greater than 1 Diopter. This will introduce confusion in interpreting corneal curvature maps.

Another important consideration in the clinical set-up is the clinically relevant inaccuracy of Placido-based corneal topography in determining the quadrafoil aberration. This could lead to false diagnosis such as ruling out the corneal shape as a cause of low vision after corneal topography measurements reveal low quadrafoil aberration in cases where the actual quadrafoil aberration is high. Point to point topography will avoid these situations.

6. Conclusion

In this paper we developed a forward ray tracing algorithm that can be used for image projection prediction and surface reconstruction in corneal topography. Consequently, it enabled the evaluation of four surface reconstruction algorithms. Two different versions of the arc step algorithm used in Placido-based corneal topography were compared to two algorithms based on the principle of forward ray tracing. It has been demonstrated that the performance of algorithms that use one-to-one correspondence of source and image points are superior to algorithms using meridional constraints even with improved features such as skew ray error correction. When no noise is present, we note that the first order correction to the skew ray error of the basic arc step algorithm gives reasonable results. The arc step algorithm with skew ray error correction performs slightly better, but not in all cases. However when noise is present the arc step algorithms are demonstrably less precise and less accurate than their one-to-one correspondence counterparts. They are inaccurate particularly in determining the correct quadrafoil aberration of the cornea. The FRT algorithm is highly accurate and very robust in all cases and should therefore be preferred over arc step based algorithms.

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